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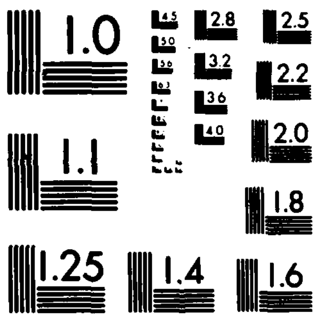

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This interim report describes the research carried out by Professors Sanjoy K. Mitter and Bernard Levy and Mr. Yehuda Avniel and Mr. Saul Gelfand during the time period March 15, 1982 and March 14, 1983, with support extended by the Air Force Office of Scientific Research under Grant AFOSR-82-0135. The principal investigator was Professor Sanjoy K. Mitter. The contract monitors were Dr. J. Bram and Dr. J. Burns of the AFOSR Directorate of Mathematical and Information Sciences. Research was carried out on the following main topics: (1) Linear and Nonlinear Filtering and related Inverse Scattering Problems; (CONTINUED)

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ITEM #20, CONTINUED: (2) Stochastic Control with Partial Observations. Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.

AFOSR-TR- 83-0982

May 1983

An Interim Technical Report of Research

Under Grant AFOSR 82-0135

for the period

March 15, 1982 to March 14, 1983

Submitted to:

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Directorate of Mathematical and Information Sciences
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ABSTRACT

This interim report describes the research carried out by Professors Sanjoy K. Mitter and Bernard Levy and Mr. Yehuda Avniel and Mr. Saul Gelfand during the time period March 15, 1982 and March 14, 1983, with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 82-0135.

The principal investigator was Professor Sanjoy Mitter. The contract monitors were Dr. J. Bram and Dr. J. Burns of the AFOSR Directors of Mathematical and Information Sciences.

Research was carried out on the following main topics:

1. Linear and Nonlinear Filtering and related Inverse Scattering Problems
2. Stochastic Control with Partial Observations.

Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.

1. Introduction

Two fundamental aspects of stochastic system theory are filtering theory and stochastic control theory. Indeed, the two aspects are interrelated in the sense that stochastic control in the presence of incomplete and uncertain information about the state of the system requires estimating the state from noisy observations on the system. This is the domain of filtering theory. Moreover, the problem of parameter identification could be considered as a special case of nonlinear filtering and the problem of optimal adaptive control (suitably formulated) can be considered as a special case of stochastic control with partial uncertain observations. One must remark that this necessitates taking a Bayesian point of view.

Stochastic Calculus of Variations could be considered as a special case of Stochastic Control. This is well known in the deterministic situation and indeed the two fields could be considered equivalent, in the sense that by appropriate transformations one can pass from one formulation to the other. This is however not so in a stochastic setting where careful distinction needs to be made between "open-loop control" (preprogrammed control) and "feedback control" (control based on past history of the observations).

~~In~~ This report ~~we~~ discusses progress made on linear and non-linear filtering theory during the grant period, March 15, 1982 to March 14, 1983. The two aspects ~~on which we have~~ of research included: are

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~~made progress are:~~

(i) Stochastic Control Interpretation of Non-linear filtering and its implication on approximate non-linear filtering, especially a rigorous analysis of the Extended Kalman filter~~.~~, and

(ii) Scattering and Inverse Scattering Interpretation of Linear Estimation and development of new algorithms for linear estimation.

We mention that the work proposed here is potentially of great benefit to the U.S. Air Force. Increasingly, it is being recognized that ad hoc techniques using linearization and perturbation methods are unsatisfactory and non-linear theory is ripe for applications. The Kalman filter has played an important role in guidance and control of aerospace vehicles. However, the extended Kalman filter which is used to handle nonlinear situations is not understood from a scientific point of view and often gives rise to incurable convergence difficulties. We expect to significantly alleviate the situation.

The control of future aircrafts and aerospace vehicles is a problem of continuing importance from the point of view of designing adaptive systems that operate reliably over a wide operating envelope. Similarly, the control of advanced jet engines, whose dynamic characteristics change rapidly with operating conditions, pose difficult problems if one

wishes to design a control system which accomplishes commanded thrust level changes rapidly, while maintaining fan and compressor stability margins. It would appear that an adequate theory of stochastic control with imperfect observations will be essential to solve these problems. Due to the tremendous increase in computing power available and decrease in costs in memory size the problem of dealing with nonlinearities is no longer the insurmountable obstacle it was. Our research is directed towards achieving this goal.

2. Research Progress

2.1 Nonlinear Filtering

The main progress we have made is related to Section 4.1.2 (Stochastic Control Interpretations of Nonlinear Filtering) of the comprehensive proposal submitted to the Air Force Office of Scientific Research in October 1981. We summarize this now.

A long-standing open problem in filtering theory has been to obtain a derivation of the Extended Kalman Filter and explain its qualitative behavior. The Extended Kalman Filter is widely used in aerospace systems and is known to function very well in many situations but is also known to exhibit divergence phenomenon in the presence of modelling errors. No satisfactory explanation of this is known to us. In the paper [1], first steps towards a rigorous derivation of the Extended Kalman Filter as well as explaining its qualitative

properties have been taken. A detailed paper jointly with Wendell Fleming of Brown University is now in preparation. To obtain this result we use the stochastic control interpretation of nonlinear filtering described in the joint paper with Wendell Felming [2], and the Morse Lemma with parameters [3].

In Section 4.1.2 of the previous proposal it was suggested that the Stochastic Control Interpretation of Nonlinear Filtering would provide the means for obtaining bounds for nonlinear filtering. A first step in this direction has been taken in the paper mentioned above. Specifically, the analog of the Fisher Information Matrix has been defined. This paper also shows how the nonlinear filtering problem is related to the identification problem.

We have also made progress on research proposed in Section 4.1.1, specifically Problems 1 and 3 described on pages 8 and 9 of the previous comprehensive proposal. Some progress has been made in this problem. In particular, it appears that the vector cubic sensor problem has a negative answer with respect to obtaining a robust form of the filter. In the scalar case, we proved [2] that this problem had a positive answer.

Problem 3 on the smoothness of densities of the Zakai equation was described as an open problem in the invited talk given by one of us in the Mexico Meeting on Nonlinear Filtering and Stochastic Control (organized by Wendell Fleming) in

January 1982. Daniel Stroock of the University of Colorado has solved this problem in as yet unpublished work (Private Communication to Mitter). The question of smoothness of the Zakai equation as a function of y (the observations) and the use of the Malliavin Calculus to obtain bounds on nonlinear filtering is being pursued in joint work with D. Ocone of Rutgers University (former student of Mitter).

2.2 Linear Estimation and Scattering Theory

The objective of our research has been the study of linear estimation problems in one or several dimensions, and the use of scattering and inverse scattering techniques to obtain efficient estimation procedures. Progress on this work has been reported in [1]-[3], and will be described in the forthcoming doctoral thesis of Y. Avniel.

In one dimension, there exists several techniques to estimate a stationary stochastic process given some observations of this process over a finite interval. The best known method, which is due to Levinson [4], exploits the Toeplitz structure of the covariance of the observed process to obtain some recursions for the linear least-squares predictor of the process. These recursions require only $O(N^2)$ operations to compute the predictor of order N. The Levinson algorithm is currently in use in many areas of application such as speech processing, geophysical signal processing and spectral estimation. However, this algorithm suffers from an important limitation: it cannot be extended to

higher dimensions, i.e., to estimate isotropic random fields.

Another estimation procedure was proposed by Krein [5], and Dym and McKean [6], [7]. In this procedure, the spectral function of the stochastic process is identified with that of a second-order differential operator associated to a vibrating string. Inverse spectral techniques [7], [8] are then used to reconstruct the mass distribution of the string from its spectral function, and this mass distribution is used in turn to solve the original estimation problem. However, because of the depth of the mathematical ideas motivating this procedure, and because the algorithm used to reconstruct the mass of the string from its spectral function is not particularly efficient, this method has only received a limited amount of attention thus far.

Our contribution to this subject has been:

1) To develop a variant of the approach of Krein and Dym and McKean, where the spectral function of the observed process is identified with that of a Schrödinger operator. The inverse spectral problem consists now in reconstructing the potential function of the Schrödinger operator from its spectral function. This can be done by using the Gelfand-Levitan [9], inverser spectral procedure. By nothing that this procedure requires the solution of equations with a Toeplitz plus Hankel structure, we have obtained a fast

algorithm for performing the potential reconstruction. This algorithm has the same efficiency, and is directly related to the Levinson recursions. It also provides an efficient solution of our original estimation problem.

2) Although our results are ultimately expressed in a language similar to that of Dym and McKean [6], [7], our estimation procedure is derived entirely from a stochastic point of view, by using elementary principles only. This requires the introduction of symmetric and antisymmetric versions of the observed process, and the solution of some associated filtering problems. These symmetric and antisymmetric processes have covariances with a Toeplitz plus Hankel structure, and in this context, the Gelfand-Levitan equations can be viewed as providing the optimum filters for the symmetric and antisymmetric filtering problems. Thus, our estimation procedure is based on simple ideas which are somewhat easier to motivate than the approach of Krein, or Dym and McKean.

3) The most significant accomplishment of our research has been, however, that we have been able to extend our estimation procedure to higher-dimensional isotropic random fields (see [2]). To do so, we follow the same approach as in the one dimensional

case, i.e., we identify the spectral function of the isotropic random field with that of a circularly (or spherically) symmetric Schrödinger operator. Then, the Gelfand-Levitan procedure can be used to reconstruct the potential from its spectral function (see [10], [11]). Furthermore, by exploiting the structure of the kernals defining the Gelfand-Levitan equation, we have obtained a fast algorithm to solve this equation. This algorithm solves the estimation problem where one is given some observations of an isotropic random field over a disk, or a sphere, and where one wants to construct the smoothed estimates of this field. The algorithm that we have obtained is the <u>first</u> fast algorithm <u>ever obtained</u> to solve a random field estimation problem. Its efficiency is compared to that of the Levinson algorithm in the 1-D case. As in the 1-D case, we can also interpret our results from a stochastic point of view by noting that the isotropic random field that we consider can be decomposed in Fourier or spherical coefficients define some 1-D random processes whose covariance has some structure which can be exploited. Then, by considering the filtering problem associated with these problems, we obtain the Gelfand-Levitan equation mentioned above, and

its solution gives the optimum filter for the Fourier or spherical coefficient estimation problem.

4) We have been able to pursue the analogy between linear estimation and quantum mechanics somewhat further, and in [3] we have obtained a stochastic interpretation of bound states, and of the Marchenko inverse scattering method of quantum mechanics. Instead of assuming that the spectral function, or that the magnitude of the scattered wave is known as in the Gelfand-Levitan approach, the Marchenko method assumes that the phase-shift of the scattered wave is observed. In the stochastic context, this problem can be viewed as one of reconstructing a minimum phase filter from its phase. This problem has received a substantial amount of attention recently [12], [13], the Marchenko method provides a fast algorithm to perform the reconstruction.

In several places in the previous proposal (for example, pages 23 and 30) it was suggested that a deeper understanding of the relation between the Scattering Theory of Lax and Phillips, the theory of Minimal Unitary Deletions due to Nagy and Foias and Linear Estimation Theory was needed. This has been undertaken in the doctoral dissertation of Y. Avniel (under the direction of

Mitter) and the main work is now complete. The relations appear to be deep and it has been shown how the scattering function induces a Wiener-Hopf operator which solves the estimation problem. By using the work of R. Howe [22] a representation of the Wiener-Hopf operator as a distribution on a Heisenberg group can be obtained. Details of this work will be available in the doctoral dissertation of Y. Avniel and in several papers in preparation by Mitter.

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2. B.C. Levy and J.N. Tsitsiklis, "An Inverse Scattering Procedure for the Linear Estimation of Two-Dimensional Isotropic Random Fields," Technical Report, LIDS, MIT, November 1982.

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1. S.K. Mitter: "Nonlinear Filtering of Diffusion Processes: A Guided Tour," in Advances in Filtering and Stochastic Control, eds. W.H. Fleming and L.G. Goristizia, Springer Lecture Notes in Control and Information Sciences, 42 (1982), 256-266.

2. W. Fleming and S.K. Mitter: "Optimal Control and Nonlinear Filtering for Nondegenerate Diffusion Processes," Stochastics, 8, pp. 63-78, 1982.

3. S.K. Mitter: "Approximations to Nonlinear Filtering," to appear in Nonlinear Stochastic Systems, eds. R.S. Bucy and J. Moura, Reidel, Dordrecht, 1983.

4. S.K. Mitter: "Lectures on Nonlinear Filtering and Stochastic Control," in Proceedings, Cortona 1981, Springer Lecture Notes in Mathematics, 972, Berlin-New York, Springer-Verlag, 1983.

5. S.K. Mitter: "Geometric Theory of Nonlinear Filtering," to appear in Outils et Méthodes Mathématiques pour l'Automatiques, Vol. 3.

6. B.C. Levy and J. N. Tsitskilis: "Linear Estimation of Stationary Stochastic Processes, Vibrating Strings, and Inverse Scattering," submitted to IEEE Trans. on Information Theory.

7. B.C. Levy and J.N. Tsitsiklis: "An Inverse Scattering Procedure for the Linear Estimation of Two-Dimensional Isotropic Random Fields," submitted to IEEE Trans. on Information Theory.

INVITED LECTURES

S.K. Mitter

Three invited lectures at the NATO Advanced Study Institute on Nonlinear Stochastic Problems, Algarve, Portugal, May 1982.

Invited Lecturer, CNR conference on Differential Inclusions and Control Theory, Udine, Italy, September 1982.

Seminar, Department of Electrical Engineering, University of California, Berkeley, October 1982.

B.C. Levy

Two Lectures, Inverse Spectral and Inverse Scattering Techniques in Linear Estimation, Stanford University, August 1982.

Lecture, Signal Processing with State-Space Models, USC Workshop on VLSI and Modern Signal Processing, Los Angeles, California, November 1982 (Paper in preparation).

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